NOTATION

c_r = dimensionless phase velocity, $c_r^*/\overline{U}$
g = acceleration of gravity
h_0 = film thickness of basic flow
N_{Re} = Reynolds number, $\overline{U}h_0/\nu$
N_{We} = Weber number, $\sigma/\rho h_0 \overline{U}^2$
N_ζ = surface tension group, $(\sigma/\rho)\ (3/g\nu^4)^{1/3}$
$\overline{U}$ = average basic flow velocity
y = cross-stream coordinate

Greek Letters

α = dimensionless complex wave number, $2\pi h_0/\lambda$
α_i = spatial amplification factor
α_r = real wave number
β = angle of plane to horizontal
λ = complex wave length
ν = kinematic viscosity
ρ = density
σ = surface tension
ω = dimensionless angular frequency, $\omega^* h_0/\overline{U}$

Superscript

$*$ = dimensional quantity

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An Analysis of Turbulent Pipe Flow for a Viscoelastic Fluid

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A note by the author (1974) showed that the friction factor for power law non-Newtonian fluids for turbulent flow in a pipe corresponds to the velocity profile slope at $y^+ = 12$. This note extends this work to an analysis for a viscoelastic fluid.

FRICTION FACTOR RATIO

The prior note showed that the wall region frequency relationship for Newtonian fluids in turbulent flow

$$\frac{u^{*2}\, t}{\nu} = 338 \tag{1}$$

is also applicable to power law non-Newtonian fluids. Elastic fluids appear to have the property of decreasing this frequency which results in drag reduction at the pipe wall. The momentum equation for the boundary layer can be written for Newtonian and elastic fluids as

$$\frac{\tau}{\rho} = 0.332\, U_{\infty N}^2 \sqrt{\frac{\nu}{U_{\infty N}x}} \tag{2}$$

and

$$\frac{\tau}{\rho} = 0.332\, U_{\infty E}^2 \sqrt{\frac{\nu_1}{U_{\infty E}x}} \tag{3}$$

Earlier work (1973) has shown that

$$x = 338\, \nu/u^* \tag{4}$$

for Newtonian fluids corresponding to the average velocity in the laminar layer of $y^+ = 2.0$. A general form of Equation (4)

$$x = \alpha\nu_2/u^* \tag{5}$$

can be used to represent frequency and laminar layer thicknesses that could occur with elastic fluids in turbulent flow. Combining Equations (2) and (4) yields

$$U^+_{\infty N} = 14.5 \tag{6}$$

and Equations (3) and (5):

$$U^+_{\infty E} = 2.08\, \alpha_E^{1/3}\, (\nu_2/\nu_1)^{1/3} \tag{7}$$

The friction factor relationship corresponding to the dimensionless shearing stress for a developing boundary layer on a flat plate provides for Newtonian fluids

$$f_N = \frac{0.036}{\sqrt{U^+_{\infty N}}} \tag{8}$$

with substitution of Equation (4) and

$$f_E = \frac{0.664}{(\alpha_E U^+_{\infty E}\, \nu_2/\nu_1)^{1/2}} \tag{9}$$

for elastic fluids with substitution of Equation (5). Combination of Equations (6), (7), (8), and (9) yields the friction factor ratio

$$\frac{f_N}{f_E} = 0.0206\, \alpha_E^{2/3}\, (\nu_2/\nu_1)^{2/3} \tag{10}$$

EXPERIMENTAL FRICTION FACTOR DATA

Seyer and Metzner (1969) present shear stress-shear rate data and relaxation time data for 0.01% ET-597 in water. This paper also shows friction factor data for this solution with Reynolds numbers of 10,000 to 200,000. Gupta, Metzner, and Hartnett (1967) present viscometric data for 0.05% ET-597 in water and friction factor data for this solution and a 0.01% solution in the Reynolds number range of 20,000 to 100,000. Data for both solutions indicate a flow behavior index of unity. Equation (10) could then be expected to apply to these data with a kinematic viscosity ratio of unity. Figure 1 shows values of α_E calculated from Equation (10) for the friction factor data represented by the range of the experimental shear stress data. The Deborah number represents the product of the relaxation time and the slope of the turbulent velocity profile $du^+/dy^+ = 0.41$ corresponding to $y^+ = 12$. Relaxation time data for the 0.05% solution were estimated by interpolation of data for 0.01% and 0.1% solutions. A Deborah number of greater than unity is observed to represent the region of elastic flow contribution.

HEAT TRANSFER

Gupta et al. report a reduction in heat transfer rate of 62% at a Reynolds number of 61,000 for the 0.05% ET-597 solution in comparison to water and that the friction factor reduction at this flow rate is 44%. The penetration model for heat transfer with Equation (1) provides heat transfer for the transition region with Newtonian fluids

$$\left(\frac{h}{\rho C_p}\right)_N = \frac{2}{\sqrt{338\pi}}\, u^*_N\, (N_{PrN})^{-1/2} \tag{11}$$

and with the general form of Equation (1) for elastic fluids

$$\left(\frac{h}{\rho C_p}\right)_E = \frac{2}{\sqrt{\alpha_E \pi}}\, u^*_E (N_{PrE})^{-1/2} \tag{12}$$

The ratio of the heat transfer coefficients for fluids with unity flow behavior index is then

$$\frac{h_N}{h_E} = \frac{u^*_N}{u^*_E}\left(\frac{\alpha_E}{338}\right)^{1/2} \tag{13}$$

Substitution of the friction factor ratio from Equation (10) for the shear velocity ratio yields

$$\frac{h_N}{h_E} = 0.0078\, \alpha_E^{5/6} \tag{14}$$

The ratio of heat transfer rates can be assumed by combination of the coefficients with the approximate thickness of the transition region. This thickness is $y_N = y_N^+\, \nu/u^*_N$ for a Newtonian fluid and $y_E = y_E^+\, \nu/u^*_E$ for an elastic fluid. If $y_N^+ = y_E^+$, the ratio of heat transfer rates is

$$\text{rate ratio} = \frac{h_N u^*_N}{h_E u^*_E} = 0.00112\, \alpha_E^{7/6} \tag{15}$$

Comparison of Equations (10) and (15) show that a friction factor reduction of 44% will correspond to a reduction in heat transfer rate of 67% if total resistance to heat transfer is in the transition region. This is in good agreement with the observed experimental data.

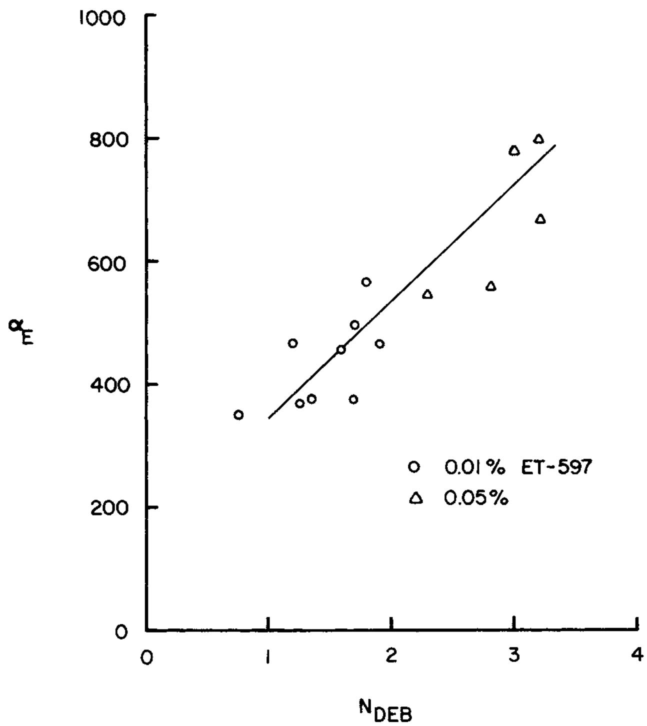

Fig. 1. Dependence of α_E on the Deborah number.

NOTATION

C_p = specific heat
f = friction factor
h = heat transfer coefficient
N_{Deb} = Deborah number
N_{Pr} = Prandtl number
t = eddy contact time
U_∞ = free stream velocity
U_∞^+ = U_∞/u^*
u = velocity in axial direction
u^* = shear velocity
u^+ = u/u^*
x = length
y = radial distance from pipe wall
y^+ = yu^*/ν

Greek Letters

α = defined by Equation (5)
ν = kinematic viscosity
ρ = fluid density
τ = shear stress at wall

Subscripts

E = elastic
N = Newtonian
1 = boundary layer
2 = bulk fluid

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